

(REVISED COURSE)

QP Code : 1037

(3 Hours)

[Total Marks : 80

- N.B. (1) Question no. 1 is compulsory.
 (2) Solve any three questions from the remaining six questions
 (3) Each questions carry equal marks.

1. (a) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ 3
 (b) Find P.I. of $(D^2 - 4D + 4) y = e^x + \cos 2x$ 3
 (c) Show that $\nabla = 1 - E^{-1}$ 3
 (d) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{1+x^2+y^2}$ 3
 (e) Solve $\left(1 + e^{x/y}\right) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$ 4
 (f) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing to polar co-ordinates 4
2. (a) Solve $y^4 dx = \left(x^{-3/4} - y^3 x\right) dy$ 6
 (b) Change the order of integration and evaluate
 $\int_0^{1/\sqrt{x}} \int_x^y \frac{y}{(1+xy)^2(1+y^2)} dy dx$ 6
 (c) (1) P.T. $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(m, n)$ 4
 (2) P.T. $\int_0^{\infty} \frac{\log(i+ax^2)}{x^2} dx = \pi\sqrt{a}$, where $a > 0$ 4
3. (a) Evaluate $\int_0^{\log 2} \int_0^{x+\log y} \int_0^{x+y+z} e^{x+y+z} dz dy dx$ 6
 (b) Find the area bounded between the parabola $x^2=4ay$ and $x^2=-4a(y-2a)$ 6

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- (c) Solve by the method of variation of parameters

8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

4. (a) Find the length of the cardioid
- $r = a(1 - \cos \theta)$
- lying outside the circle
- $r = a \cos \theta$

6

(b) Solve $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$

6

- (c) Using R.K. Method of fourth order, solve.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \text{ given } y(0) = 1 \text{ at } x = 0.2, 0.4$$

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5. (a) Solve
- $x \sin x dy + (x y \cos x - y \sin x - 2) dx = 0$

(b) Solve $\frac{dy}{dx} = 2 + \sqrt{xy}$ with $x_0 = 1.2, y_0 = 1.6403$ by modified Euler's method, for $x = 1.4$ correct to 4-decimal places, (taking $h = 0.2$)

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6

(c) Evaluate $\int_0^6 x f(x) dx$ by

(a) Trapezoidal rule

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(b) Simpson's 1/3rd rule

using the following table

x	0	1	2	3	4	5	6
f(x)	0.146	0.161	0.176	0.190	0.204	0.217	0.230

6. (a) The charge Q on the plate of a condenser of Capacity C charged through a resistance R by a steady voltage V satisfies the differential equation

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$$R \frac{dQ}{dt} + \frac{Q}{c} = V, \text{ If } Q = 0 \text{ at } t = 0, \text{ show that } i = \frac{V}{R} e^{-t/RC} \therefore i = \frac{dQ}{dt}$$

- (b) Evaluate
- $\iint_A x^2 dx dy$
- where A is the region in the first quadrant bounded by the hyperbola
- $xy = 16$
- and the lines
- $y = x, y = 0$
- and
- $x = 8$
- .

6

- (c) Find the volume of the tetrahedron bounded by the planes,
- $x = 0, y = 0, z = 0$
- and
- $x + y + z = a$

8