Duration – 3 Hours **Total Marks: 80**

N.B. 1. Question No. 1 is compulsory.
2. Attempt any **THREE** questions out of remaining **FIVE** questions.
3. Figures to right indicate full marks.

1) a) Solve $2\left(x^2\sqrt{y}+1\right)y\,dx+\left(x^2\sqrt{y}+2\right)x\,dy=0$ (4)

b) Find the particular integral of $(D-3)y=x$ (3)

c) Evaluate $\int_0^{\infty} e^{-x^2}\,dx$ (3)

d) Sketch the region of integration $I=\int_1^4\int_0^{\sqrt{x}}\frac{3}{2}e^{\left(y/\sqrt{x}\right)}dy\,dx$ (3)

e) Prove that $E=1+\Delta=e^{hD}$ (3)

f) Using Euler's method find the approximate value of y, where $\frac{dy}{dx}=\frac{y-x}{\sqrt{xy}}$ (4)

and y(1) = 2 when x = 1.5 in five steps taking h=0.1

2 a) Solve $\frac{dy}{dx}+y=y^2(\cos x-\sin x)$ (6)

b) Show that $\int_0^{\infty}\frac{\tan^{-1}ax}{x\left(1+x^2\right)}dx=\frac{\pi}{2}\log(1+a)$. Hence evaluate $\int_0^{\infty}\frac{\tan^{-1}x}{x\left(1+x^2\right)}dx$ (6)

c) Change to polar and evaluate $I=\int_0^{a}\int_y^{a+\sqrt{a^2-y^2}}\frac{dx\,dy}{\left(4a^2+x^2+y^2\right)^2}$ **(8)**

3 a) Given that $\int_0^{\infty}\frac{x^{p-1}}{1+x}dx=\frac{\pi}{\sin p\pi}$ .P.T $\Gamma(p)\Gamma(1-p)=\frac{\pi}{\sin p\pi}$ $(0<p<1)$ (6)

b) Evaluate $\iiint_V\frac{dx\,dy\,dz}{\left(1+x^2+y^2+z^2\right)^2}$ where V is the volume in the first octant. (6)

c) Solve by method of variation of parameters $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ **(8)**

4 a) Evaluate $I=\int_0^{\pi} 2d\theta \int_0^{a(1+\cos\theta)} r\,dr \int_0^{h}\left[1-\frac{r}{a(1+\cos\theta)}\right]dz$ (6)

b) Solve $(D^3+2D^2+D)y=e^{3x}\ x^2\ +\sin^2 x$ (6)

c) Using fourth order Runge-Kutta method, solve numerically **(8)**
$\frac{dy}{dx}=x^2+y^2$ with the conditions x = 1, y = 1.5 in the interval (1, 1.2) with h = 0.1 correct to 4 decimals.

5 a) The density at any point of a cardioid $r=a(1+\cos\theta)$ varies as the square of its distance from its axis of symmetry. Find its mass. (6)

b) An equation in the theory of stability of an aeroplane is (6)
$\frac{dv}{dt}=g\cos\alpha-kv$ v being velocity and g, k being constants. It is observed that at time t = 0, the velocity v = 0. Solve the equation.

c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Trapezoidal Rule, (ii) Simpson's $(1/3)^{rd}$ Rule and (iii) Simpson's $(3/8)^{th}$ Rule. Also find the error. **(8)**

6 a) Solve $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=6x$ (6)

b) For the curve $x=a(2\cos t-\cos 2t)$, $y=a(2\sin t-\sin 2t)$, find the length of the arc of the curve measured from $t=0$ to any point (6)

c) Find the volume cut off from the paraboloid $x^2+\frac{1}{4}y^2+z=1$ by the plane z = 0 **(8)**

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