Duration – 3 Hours **Total Marks : 80**

(1) N.B.:- Question no 1 is compulsory.

(2) Attempt any THREE questions out of remaining FIVE questions.

1)a) Evaluate $\int_0^{\infty} \frac{dx}{(a^2 + x^2)^5}$ (4)

b) Find the particular integral of $(D + 2)y = x^2$ (3)

c) Solve $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$ (3)

d) Express the following integral in polar co-ordinates: (4)

$$I = \int_0^4 \int_y^{4+\sqrt{16 - y^2}} f(x, y)\, dxdy$$

e) Prove that $E = 1 + \Delta$ (3)

f) Evaluate $I = \int_0^{\pi/2} \int_0^{3(1-\cos t)} x^2 \sin t \, dx\, dt$ (3)

2 a) Solve $\frac{dy}{dx} - xy = y^2 e^{-\left(\frac{x^2}{2}\right)} \log x$. (6)

b) Change the order of integration and evaluate $I = \int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y\, dy\, dx}{\sqrt{(2 - x^2)(1 - x^2 y^2)}}$ (6)

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c) Evaluate $\int_0^{\pi} \frac{dx}{a + b\cos x}$ a > 0, |b| < a. Hence show that (8)

$$\int_0^{\pi} \frac{dx}{(5 + 4\cos x)^2} = \frac{-4\pi}{27}$$

3 a) Evaluate $I = \int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\, dx\, dy\, dz$ (6)

b) Find the area between the circles $x^2 + y^2 - 4ax = 0$ and $x^2 + y^2 - 2ax = 0$ (6)

c) Solve $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = \sin \log x$ (8)

4 a) Find the total length of the curve $x = a e^{\theta} \sin \theta$, $y = a e^{\theta} \cos \theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ (6)

b) Solve $(D^2 - 3D + 2) y = \frac{1}{e^{(e-x)}} + \cos\left(\frac{1}{e^x}\right)$ (6)

c) Use Runge-Kutta method of fourth order , compute y(0.2) given $y' + y + xy^2 = 0, y(0) = 1$ by taking h = 0.1 correct to 4 decimal point. (8)

5 a) State duplication formula and prove that $\overline{\left|\frac{1}{4}\right.} \cdot \overline{\left|\frac{3}{4}\right.} = \sqrt{2}\ \pi$ (6)

b) Using Taylor's series method, obtain the solution of the differential equation $y' = y - xy$, $y(0) = 1$ (6)

c) Find the volume bounded by the paraboloid $z = 4 - x^2 - \frac{y^2}{2}$ and the plane $z = 0$. (8)

6 a) A chain coiled up near the edge of a smooth table starts to fall over the edge. The velocity v when a length x has fallen is given by $x v \frac{dv}{dx} + v^2 = gx$. Show that $v = 8\sqrt{x/3}$ (6)

b) Find the mass of a plate I the form of a cardioid $r = a(1 - \cos \theta)$ if the density at any point of the plate varies as its distance from the pole. (6)

c) Evaluate $\int_{-3}^{3} x^4\, dx$, using (i) Trapezoidal Rule (ii) Simpson's (1/3)rd rule. Compare it with exact solution. (8)
