

Duration – 3 Hours

Total Marks : 80

(1) N.B.:- Question no 1 is compulsory.

(2) Attempt any THREE questions out of remaining FIVE questions.

Q.1) a) Solve $\frac{dy}{dx} = \frac{a^2 - 2xy - y^2}{(x+y)^2}$ (4)

b) Solve $(D^3 - 3D^2 + 4)y = 0$ (3)

c) Evaluate $\int_0^{\infty} e^{-\left(x^2/4\right)} dx$ (3)

d) Express the following integral in polar co-ordinate (4)

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} f(x,y) dx dy$$

e) Prove that $E = 1 + \Delta = e^{tb}$ (3)

f) Evaluate $I = \int_0^{\pi/2} \int_{\pi/2}^{\pi} \cos(x+y) dy dx$ (3)

Q.2 a) Solve $\frac{dz}{dx} + \frac{z}{x} \log z = \frac{z}{x^2} (\log z)^3$ (6)

b) Change the order of integration and evaluate $I = \int_0^1 \int_{x^2}^{2-x} \frac{x}{y} dy dx$ (6)

c) Show that $\int_0^{\infty} \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx = \frac{\pi}{2} \log \left(\frac{a}{b} \right)$ (8)

Q.3 a) Evaluate $I = \int_0^2 \int_0^y \int_{x-y}^{x+y} (x+y+z) dx dy dz$ (6)

b) Find the mass of a plate in the form of a cardioid $r = a(1 - \cos \theta)$ (6)
if the density at any point of the plate varies as its distance from the plate.

c) Solve $(2x+1)^2 \frac{d^2 y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = x^2$ (8)

Q. 4 a) Show that the length of the curve $x = a e^{\theta} \sin \theta$ $y = a e^{\theta} \cos \theta$ from (6)

$$\theta = 0 \text{ to } \theta = \pi/2$$

b) Solve $\frac{d^2 y}{dx^2} - y = \cos x \cosh x + a^x$ (6)

c) Using fourth order Runge-Kutta method, solve numerically, the (8)

differential equation $\frac{dy}{dx} = x^2 + y^2$ with the given condition $x = 1$,
 $y = 1.5$ in the interval (1, 1.2) with $h = 0.1$

Q. 5 a) Use method of variation of parameters to solve (6)

$$\frac{d^2 y}{dx^2} + y = 3x - 8 \cot x,$$

b) Using Taylor's series method, obtain the solution of (6)

$\frac{dy}{dx} = y - xy$, $y(0) = 2$. Find the value of y for $x = 0.1$ correct to four decimal places

c) Evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$ by using (i) Trapezoidal Rule, (ii) Simpson's (1/3)rd Rule and (iii) Simpson's (3/8)th Rule. Compare the result with exact solution. (8)

Q. 6 a) In a circuit of resistance R , self inductance L , the current i is given (6)

by $L \frac{di}{dt} + Ri = E \cos pt$ where E and p are constants. Find the current i at time 't'

b) Find the area bounded by the parabola $y = 4x - x^2$ and the line $y = x$ (6)

c) Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. (8)
