

(3 Hours)

QP Code : 5801
[Total Marks : 80

- N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Evaluate $\int_0^2 x^2 (2-x)^3 dx$ [3]
(b) Solve $\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0$ [3]
(c) Prove that $E = 1 + \Delta$ [3]
(d) Solve $\left[y \left(1 + \frac{1}{x} \right) + \cos y \right] dx + (x + \log x - x \sin y) dy = 0$ [3]
(e) Change to polar coordinates and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2) dy dx$ [4]
(f) Evaluate $\int_0^1 \int_0^x xy dy dx$ [4]
2. (a) Solve $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$ [6]
(b) Change the order of integration and evaluate $\int_0^2 \int_{\sqrt{2x}}^2 \frac{y^2 dx dy}{\sqrt{y^4 - 4x^2}}$ [6]
(c) Prove that $\int_0^{\pi/2} \frac{\log(1+a \sin^2 x)}{\sin^2 x} dx = \pi [\sqrt{a+1} - 1]$ $a > -1$ [8]
3. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{(x+y+z+1)^3} dz dy dx$ [6]
(b) Find by double integration the area enclosed by the curve $9xy = 4$ and the line $2x + y = 2$ [6]

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- (c) Using method of Variation of Parameter solve $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$ [8]
4. (a) Find the perimeter of the cardioid $r = a(1 + \cos\theta)$ [6]
 (b) Solve $(D^2 + 4)y = \cos 2x$ [6]
 (c) Apply Runge-kutta Method of fourth order to find an approximate value of y for $\frac{dy}{dx} = \frac{1}{x+y}$ with $x_0 = 0, y_0 = -1$ at $x = 1$ taking $h = 0.5$ [8]
5. (a) Solve $(y - xy^2)dx - (x + x^2y)dy = 0$ [6]
 (b) Using Taylor Series Method obtain the solution of following differential equation $\frac{dy}{dx} = 1 + y^2$ with $y_0 = 0$ when $x_0 = 0$ for $x = 0.2$ [6]
 (c) Find the approximate value of $\int_0^6 e^x dx$ by i) Trapezoidal Rule, ii) Simpson's $1/3^{\text{rd}}$ Rule, iii) Simpson's $3/8^{\text{th}}$ Rule [8]
6. (a) A resistance of 100 ohms and inductance of 0.5 henries are connected in series with a battery of 20 volts. Find the current at any instant if the relation between L, R, E is $L \frac{di}{dt} + Ri = E$. [6]
 (b) $\iint y dx dy$ over the area bounded by the $x = 0, y = x^2, x + y = 2$ [6]
 (c) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$ [8]