Time: 3 hours Marks: 80

N.B 1) Question **No. 1** is **Compulsory**.

2) **Answer** any **three** questions from remaining questions.

3) Figures to the right indicate full marks.

Q.1 a) Evaluate $\int_0^{\infty} \frac{e^{-x^3}}{\sqrt{x}}\, dx$. 3

b) Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 2$. 3

c) Solve $(D^2 + D)y = e^{4x}$. 3

d) Evaluate $\int_0^1 \int_{x^2}^{x} xy(x + y)\, dy dx$. 3

e) *Solve* 4

$$(4x + 3y - 4)dx + (3x - 7y - 3)dy = 0.$$

f) Solve $\frac{dy}{dx} = 1 + xy$ with initial condition $x_0 = 0, y_0 = 0.2$ by Taylors series method. Find the approximate value of y for x=0.4 (step size 0.4). 4

Q.2 a) Solve $\frac{d^2y}{dx^2} - 16y = x^2 e^{3x} + e^{2x} - cos3x + 2^x$. 6

b) Show that $\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x} dx = \pi \sin^{-1} a$ $0 \le a \le 1$. 6

c) Change the order of integration and evaluate $\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dx dy$. 8

Q.3 a) Evaluate $\iiint (x + y + z)\, dxdydz$ over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ *and* $x + y + z = 1$. 6

b) Find the mass of the lamina bounded by the curves $y = x^2 - 3x$ and $y = 2x$ if the density of the lamina at any point is given by $\frac{24}{25}xy$. 6

c) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 3y = \frac{logx \cdot cos(logx)}{x}$. 8

Q.4 a) Find by double integration the area bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$. 6

b) Solve $\frac{dy}{dx} + xsin2y = x^3 cos^2 y$. 6

c) Solve $\frac{dy}{dx} = x^3 + y$ with initial conditions $y(0) = 2$ at x=0.2 in steps of h=0.1 by Runge Kutta method of fourth order. 8

Q.5 a) Evaluate $\int_0^1 x^5 sin^{-1}x\, dx$ and find the value of $\beta\left(\frac{9}{2}, \frac{1}{2}\right)$. 6

b) In a circuit containing inductance L, resistance R, and voltage E, the current i is given by $L\frac{di}{dt} + Ri = E$. Find the current i at time t if at t=0,i=0 and L,R, E are constants. 6

c) Evaluate $\int_0^6 \frac{dx}{1+3x}$ by using i) Trapezoidal ii) Simpsons (1/3)rd and iii) Simpsons (3/8)th rule. 8

Q.6 a) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$. 6

b) Change to polar coordinates and evaluate $\int_0^1 \int_0^x (x + y)dydx$. 6

c) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. 8

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