Paper / Subject Code: 58501 / Applied Mathematics - I.

F.E. SEM - I / CREDIT BASE / MAY 2019 / 10.05.2019

(3 hours) K.G.C.E. EXAM KARJAT, RAIGAD **Total marks:80**

N.B.: (1) Question **No. 1** is **compulsory.**
(2) Attempt **any Three** from remaining.

Q1 a) If $\tanh x = 1/2$ then find value of x and $\sinh 2x$ [3]

b) If $u = \log(x^2 + y^2)$ Find $\frac{\partial^2 u}{\partial x \partial y}$ [3]

c) If $x = u - uv$, $y = uv - uvw$, $z = uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [3]

d) Using Maclaurin's series , Prove $log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} + \cdots$ [3]

e) Check if the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$ is unitary [4]

f) Find n^{th} derivative of $\frac{2}{(x-1)(x-2)(x-3)}$ [4]

Q2. a) Solve $x^5 = 1 + i$ and find the continued product of the roots. [6]

b) Reduce the matrix $A = \begin{bmatrix} 3 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ to the normal form and find its Rank [6]

c) State and Prove Euler's theorem for two variables hence find value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \frac{xy}{x+y}$ [8]

Q3 a) Investigate for what values of λ and μ the equations [6]
$x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have
i) No solution ii) Unique solution iii) Infinite solutions

b) Examine the function for its extreme values [6]
$f(x, y) = y^2 + 4xy + 3x^2 + x^3$

c) If $tan\ (\alpha + i\beta) = sin\ (x + iy)$ then Prove $\frac{tan\ x}{tanh\ y} = \frac{sin\ 2\alpha}{sinh\ 2\beta}$ [8]

Q4 a) If $x = u\cos v$, $y = u \sin v$ then [6]
Prove $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$

b) Prove that $log\left(\frac{sin(x+iy)}{sin(x-iy)}\right) = 2i tan^{-1}(cot\ x\ tanh\ y)$ [6]

c) Solve by Gauss Jordan method [8]
$2x + 3y + 4z = 1$, $x + 5y + z = 1$, $x + y + 6z = 5$

Q5. a) Prove $\cos^6\theta - \sin^6\theta = \frac{1}{32}[\cos 6\theta + 15 cos\ 2\theta]$ [6]

b) Evaluate $\lim_{x\to 0}\left[\frac{x - sin\ x}{x^3}\right]$ [6]

c) If $y = \cos(m \sin^{-1} x)$ then [8]
prove that $(1 - x^2)y_{n+2} - (2n + 1)x\ y_{n+1} + (m^2 - n^2)y_n = 0$

Q6 a) Check if the following vectors [6]
$X_1 = [3,1,2,1], X_2 = [4,6,2,-4],\ X_3 = [-6,0,-3,-4]$
$X_4 = [1,0,2,1]$ are linear dependent hence find the relation between them if any.

b) If $f\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$ then [6]
prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$

c) Fit a second degree parabola $y = ax^2 + bx + c$ to the following data [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9
