QP Code : **1002**

(REVISED COURSE)

(3 Hours) [Total Marks : 80

N. B. : (1) Question No. 1 is **compulsory**.
  (2) Answer any **three** questions from remaining.
  (3) Assume suitable data if necessary.

1. (a) If $\tan \frac{x}{2} = \tanh \frac{u}{2}$ then S.T. 3

  $u = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$

  (b) If $u = x^y$ find $\dfrac{\partial^3 u}{\partial x \, \partial y \, \partial x}$ 3

  (c) If $ux = yz$, $vy = zx$, $wz = xy$ 3

  find $J\left(\dfrac{u, v, w}{x, y, z} \right)$

  (d) If $y = (x-1)^n$ then P.T. $y + \dfrac{y_1}{1!} + \dfrac{y_2}{2!} + \dfrac{y_3}{3!} + \dots \dfrac{y_n}{n!} = x^n$ 3

  (e) P.T. $\sinh x = X + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} +$ 4

  (f) Express the matrix A as sum of Hermition and skew Hermition matrix where 4

  $\begin{bmatrix} 3i & -i+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$

2. (a) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ 6
  (b) Reduce the matrix A to normal form and hence find its rank where 6

  $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

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(c) State and prove Euler's theorem for three variables and hence find   8

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial x} \text{ where}$$

$$u = \frac{x^3 y^3 z^3}{x^3 + y^3 + z^3}$$

3. (a) Solve the following system of equations   6
   $$2x - 2y - 5z = 0$$
   $$4x - y + z = 0$$
   $$3z - 2y + 3z = 0$$
   $$x - 3y + 7z = 0$$

   (b) Find the maximum and minimum values   6
   of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

   (c) Separate into real and imaginary parts of $\tanh^{-1}(x + iy)$ 8

4. (a) If $u = 2xy$, $v = x^2 - y^2$ and   6
   $x = r\cos\theta$, $y = r\sin\theta$ then find

   $$\frac{\partial(u_1 v)}{\partial(\partial_1 \theta)}$$

   (b) If $i^{i^{i^{\cdots^{\infty}}}} = A + i B$, prove that   6

   $$\left(\frac{\pi A}{2}\right) = \frac{B}{A} \quad \text{and } A^2 + B^2 = e^{-\pi B}$$

   (c) Solve by crouts method the system of equations   8
   $$3x + 2y + 7z = 4$$
   $$2x + 3y + z = 5$$
   $$3x + 4y + z = 7$$

5. (a) By using De Moiverse thm   6

   Express $\frac{\sin 7\theta}{\sin \theta}$ in powers of $\sin\theta$ only.

   (b) By using Taylor's series expand $\tan^{-1} x$ in positive powers of $(x-1)$ upto   6
   first four non-zero terms.

   (c) If $y = \sin[\log(x^2 + 2x + 1)]$ prove that   8
   $(x + 1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2 + 4)y_n = 0$

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6. (a) Determine linear depondance or independance of vectors   6
        $x_1 = [1, 3, 4, 2]$ $x_2 = [3, -5, 2, 6]$
        $x_3 = [2, -1, 3, 4]$ and if dependent find the relation between them.

    (b) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$ prove that   6

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4\sqrt{u^2 + v^2}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$$

    (c) (i) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin x.\sin^{-1} x - x^2}{x^6}$ 4

        (ii) Fit straight line to the following data   4
        $(x, y) = (-1, -5), (1, 1), (2, 4), (3, 7), (4, 10)$
        Estimate y when x = 7

Course: F.E. (REV.) (ALL BRANCHES) (CBSGS) (SEM-I)(Prog-569)

Q.P Code: 1002

Correction:

Q. No.(4)(a)

Read AS:

$$\frac{\partial(u,v)}{\partial(r,\theta)}$$

Instead of:

$$\frac{\partial(u_1 v)}{\partial(\partial_1 \theta)}$$

Q. no. (3)(A) Equation no. 3

Read AS:

$$3x - 2y + 3z = 0$$

Instead of:

$$3z - 2y + 3z = 0$$

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Note : Take printouts and submit them to all concerned students.

Course: F.E. (REV.) (ALL BRANCHES) (CBSGS) (SEM-I)(Prog-569)

Q.P Code: 1002

Correction:

Q. No.(2)(c)

Read AS:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$$

Instead of:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial x}$$

Q. No.(4)(b)

Read AS

$$\tan\left(\frac{\pi A}{2}\right) = \frac{B}{A}$$

Instead of:

$$\left(\frac{\pi A}{2}\right) = \frac{B}{A}$$

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