

[Time: 3 Hours]

[ Marks:80]

Please check whether you have got the right question paper.

- N.B: 1. Question 1 is compulsory.  
2. Attempt any three questions from Q.2 to Q.6.

**Q1** (a) Prove  $\cosh^5 x = \cosh 5x + 5 \cosh 3x + 10 \cosh x$  (3)

(b) If  $u = \log(\tan x + \tanh x)$   
Prove  $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2$  (3)

(c) If  $u = \frac{yz}{x}$ ,  $v = \frac{zx}{y}$ ,  $w = \frac{xy}{z}$  Show that  $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$  (3)

(d) Express the following matrix as sum of symmetric and skew symmetric matrix. (3)

$$A = \begin{pmatrix} 2 & 2+i & 3 \\ -2+i & 0 & 4i \\ -i & 3-i & 1-i \end{pmatrix}$$

(e) Show that  $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6}$  (4)

(f) If  $y = \frac{x^2}{(x-1)(x-2)}$  Find  $y_n$  (4)

**Q2** (a) Solve the Equation  $x^4 - x^3 + x^2 - x + 1 = 0$  (6)

(b) Reduce the following Matrix to the Normal form and hence find the rank of the matrix (6)

$$A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$$

(c) If  $u = \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos^{-1} \left( \frac{xy + yz}{\sqrt{x^2 + y^2 + z^2}} \right)$  (8)

Find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$

**Q3** (a) (a) Investigate for what values of  $\lambda$  and  $\mu$  the system of equations  $x+2y+3z=4$ ,  $x+3y+4z=5$ ,  $x+3y+\lambda z=\mu$ . (6)  
 have 1) unique solution, 2) Infinite solutions, 3) No solution

(b) Find the Extreme values of  $f(x,y)=xy+a^3\left(\frac{1}{x}+\frac{1}{y}\right)$  (6)

(c) Separate into real and imaginary parts of  $\tan^{-1}(e^{i\theta})$  (8)

**Q4** (a) If  $u^2 + xv^2 = x + y$ ,  $v^2 + yu^2 = x - y$  Find  $\frac{\partial u}{\partial x} \frac{\partial v}{\partial y}$  (6)

(b) If  $\log \cos(x+iy) = a+ib$  Prove  $2e^{2a} = \cosh 2y + \cos 2x$  (6)

(c) Solve the following Equations by Gauss Seidel method Up to four iterations. (8)

$$4x-2y-z=40, x-6y+2z=-28, x-2y+12z=-86$$

**Q5** (a) Using De Moivre's theorem Prove (6)  
 $\cos^7 \theta = \frac{1}{2^6} (\cos 7\theta + 7 \cos 5\theta + 21 \cos 3\theta + 35 \cos \theta)$

(b) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \cot^2 x \right)$  (6)

(c) If  $y = \sin(m \sin^{-1} x)$  Prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$  (8)  
 And hence find  $y_3(0)$ .

**Q6** (a) (a) Show the following vectors are linearly dependent and find the relation between them. (6)  
 $[2, -1, 3, 2], [1, 3, 4, 2], [3, -5, 2, 2]$ .

(b) If  $z=f(x,y)$  where  $x = u \cosh v$ ,  $y = u \sinh v$  Prove (6)  
 $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2} \left(\frac{\partial z}{\partial v}\right)^2$

(c) Fit the curve of the form  $y = ab^x$  to the following data. (8)

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

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