FE, semester I, Choice base, FH2018

Q. P. Code : 38779

App. Maths.-I

14/05/18

(3 Hours) **(Total Marks: 80)**

Please check whether you have the right question paper.

N.B.: 1) Questions **No. 1** is **compulsory**.

2) **Answer any three** from remaining **five** questions.

1. a) If $\tan\frac{x}{2} = \tan h\frac{u}{2}$, show that $u = \log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$. **[3]**

b) Prove that the following matrix is orthogonal & hence find A^{-1}, **[3]**

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

c) State Euler's theorem on Homogeneous function of two variables & if **[3]**

$u = \frac{x+y}{x^2+y^2}$ then evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

d) If $u = r^2\cos 2\theta$, $v = r^2\sin 2\theta$. Find $\frac{\partial(u, v)}{\partial(r,\theta)}$. **[3]**

e) Find the n^{th} derivative of $\cos 5x \cdot \cos 3x \cdot \cos x$. **[4]**

f) Evaluate : $\lim_{x\to 0}\left(\frac{2x+1}{x+1}\right)^{\frac{1}{x}}$. **[4]**

2. a) Solve $x^4 - x^3 + x^2 - x + 1 = 0$. **[6]**

b) If $y = e^{\tan^{-1}x}$. Prove that **[6]**

$(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$.

c) Examine the function $f(x, y) = xy(3 - x - y)$ for extremes values & **[8]**

also find maximum and minimum values of $f(x, y)$.

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3\. a) Investigate for what values of λ & μ the equations $x+y+z=6$; $x+2y+3z=10$; $x+2y+\lambda z=\mu$ have **[6]**

i) no solution,

ii) a unique solution,

iii) infinite no. of solutions.

b) If $u=f\left(\dfrac{y-x}{xy},\dfrac{z-x}{xz}\right)$, show that $x^2\dfrac{\partial u}{\partial y}+y^2\dfrac{\partial u}{\partial y}+z^2\dfrac{\partial u}{\partial y}=0$. **[6]**

c) Prove that $\log\left(\dfrac{a+ib}{a-ib}\right)=2i\tan^{-1}\left(\dfrac{b}{a}\right)$ & **[8]**

$$\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]=\frac{a^2-b^2}{a^2+b^2}.$$

4\. a) If $u=\sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Prove that **[6]**

$$x^2u_{xx}+2xy\,u_{xy}+y^2u_{yy}=\frac{-\sin u\cos 2u}{4\cos^3 u}.$$

b) Using encoding matrix $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$; encode & decode the message 'ALL IS WELL'. **[6]**

c) Solve the following equations by Gauss Seidal method : **[8]**

$$10x_1+x_2+x_3=12$$
$$2x_1+10x_2+x_3=13$$
$$2x_1+2x_2+10x_3=14$$

5\. a) If $u=e^{xyz}\,f\left(\dfrac{xy}{z}\right)$ where, $f\left(\dfrac{xy}{z}\right)$ is an arbitrary function of $\dfrac{xy}{z}$, **[6]**

Prove that $x\dfrac{\partial u}{\partial x}+z\dfrac{\partial u}{\partial z}=y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=2xyz\cdot u$.

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b) Prove that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$. [6]

c) i) Prove that $\log(\sec x) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \ldots\ldots$ [4]

ii) Expand $(2x^3 + 7x^2 + x - 1)$ in powers of $(x - 2)$. [4]

6. a) Prove that $\sin^{-1}(\operatorname{cosec}\theta) = \frac{\pi}{2} + i\log\left(\cot\frac{\theta}{2}\right)$. [6]

b) Find non-singular matrices P & Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is reduced to normal form. Also find its rank. [6]

c) Obtain the root of $x^3 - x - 1 = 0$ by Regula Falsi Method (Take three iterations). [8]

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