F-E.(Sem-I)/Choice base/A-Maths-I/May 17

Q.P. Code :16227

16.05.2017

[Time: Three Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B: 1. Question.No.1 is compulsory.
2. Answer any three questions from remaining.

K.G.C.E. EXAM KARJAT, RAIGAD

Q.1 a) Prove that $\tanh^{-1}(\sin\theta) = \cosh^{-1}(\sec\theta)$ 03

b) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary 03

c) If $x = uv$ & $y = {}^{u}/_{v}$ prove that $JJ^1 = 1$ 03

d) If $Z = \tan^{-1}\left(\frac{x}{y}\right)$ where $x = 2t$, $y = 1- t^2$, prove that $\frac{dZ}{dt} = \frac{2}{1+t^2}$ 03

e) Find the n^{th} derivative of $(\cos 5x.\cos 3x.\cos x)$ 04

f) Evaluate $\lim_{x\to 0}(x)^{\frac{1}{1-x}}$ 04

Q.2 a) Find all values of $(1+ i)^{\frac{1}{3}}$ & show that their continued product is $(1 + i)$ 06

b) Find non-singular matrices P&Q such that PAQ is in normal form where 06

$A=\begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

c) Find the maximum & minimum values of $f(x,y) = x^3 + 3xy^2-15x^2-15y^2+72x$ 08

Q.3 a) If $u = f\left(\frac{y-x}{xy}, \frac{Z-x}{xZ}\right)$, show that 06

$x^2\frac{\partial u}{\partial z} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial Z} = 0$

b) Using encoding matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, encode & decode the message 'MUMBAI'. 06

c) Prove that $\log\left[\tan\left(\frac{\pi}{4} + \frac{ix}{2}\right)\right] = i\tan^{-1}(\sinh x)$ 08

Q.4 a) Obtain $\tan 5\theta$ in terms of $\tan\theta$ & show that $1 - 10\tan^2\frac{\pi}{10} + 5\tan^4\frac{\pi}{10} = 0$ 06

b) If $y = e^{\tan^{-1}x}$, prove that 06

$(1+x^2)y_{n+2} + [2(n+1)x-1]\, y_{n+2} + n(n+1)y_n = 0$

c) i. Express $(2x^3 + 3x^2 - 8x + 7)$ in terms of $(x-2)$ using Taylor's theorem. 04

ii. Prove that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + - - - - -$ 04

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Q.5 a) If $z = x^2 \tan^{-1}(\frac{y}{x}) - y^2 \tan^{-1}(^x/_y)$ — 06

Prove that $\frac{\partial^2 z}{\partial y\, \partial x} = \frac{x^2-y^2}{x^2+y^2}$

b) Investigate for what values of λ & μ the equations, — 06

$$2x + 3y + 5Z = 9$$
$$7x + 3y - 2Z = 8$$
$$2x + 3y + \lambda Z = \mu$$

Have 1) no solution

2) a unique solution

3) an infinite no. of solutions

c) Using Newton Raphson method, find approximate root of $x^3 - 2x - 5 = 0$ (correct to three places of decimals.) — 08

Q.6 a) Find tanhx if 5 sinhx – coshx = 5 — 06

b) If $u = \sin^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$ prove that — 06

i. $xu_x + yu_y = \frac{1}{2}\tan u$

ii. $x^2u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \frac{-\sin u \cos 2u}{4\cos^3 u}$

c) Solve the following systems of equations by Gauss-seidel method, — 08

$$20x + y - 2Z = 17$$
$$3x + 20y - Z = -18$$
$$2x - 3y + 20Z = 25$$

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