(Time: 3 hours)   Total Marks: 80

N.B. (1) Question no. 1 is Compulsory
         (2) Solve any three from the remaining.

Q.1) a) Prove that $\left(1 + i\sqrt{3}\right)^8 + \left(1 - i\sqrt{3}\right)^8 = -2^8$ (3)

   b) If $A = \frac{1}{3}\begin{pmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{pmatrix}$ is orthogonal find a,b,c.   (3)

   c) $z^3 + xy - y^2z = 6$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where z is an implicit
      function of x and y.   (3)

   d) If $u = e^x cosy, v = e^x siny$ find $\frac{\partial(u,v)}{\partial(x,y)}$.   (3)

   e) Find the nth derivative of $y = \frac{x^2+4x+1}{x^3+2x^2-x-2}$ (4)

   f) Find a,b if $\lim\limits_{x->0} \frac{asinhx+bsinhx}{x^3} = \frac{5}{3}$ by L'Hospitals Rule.   (4)

Q.2) a) Find the roots common to $x^4 + 1 = 0$ and $x^6 - i = 0$ (6)
   b) If $y = sin^{-1}x$ Prove That
      $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$ also find $y_9(0)$ (6)
   c) Discuss the maxima and minima of
      $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (8)
      Hence find maximum and minimum value of $f(x,y)$.

Q.3) a) Find the values of k for which the equations   (6)
      $x + y + z = 1, x + 2y + 3z = k, x + 5y + 9z = k^2$ have a solution,
      solve them for these values of k.
   b) If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$, Prove that   (6)
      $x\frac{\partial\emptyset}{\partial x} + y\frac{\partial\emptyset}{\partial y} + z\frac{\partial\emptyset}{\partial z} = u\frac{\partial\emptyset}{\partial u} + v\frac{\partial\emptyset}{\partial v} + w\frac{\partial\emptyset}{\partial w}$
      where $\emptyset$ is the function of x,y,z.
   c) If $tan(\alpha + i\beta) = cos\theta + isin\theta$ Prove that   (8)
      $\alpha = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ & $\beta = \frac{1}{2}logtan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

Q.4) a) If $z = e^{x/y} + log(x^3 + y^3 - x^2y - xy^2)$, Find the value of   (6)
      $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2}$.
   b) Using encoding matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ encode and decode the message   (6)
      NOW*STUDY
   C) Solve the following equations by Gauss Jacobi's Iteration method   (8)
      $15x + 2y + z = 18, 2x + 20y - 3z = 19, 3x - 6y + 25z = 22$

Q.5) a) Prove that the general value of $(1 + itan\alpha)^{-i}$ is   (6)
$$e^{2m\pi+\alpha}[cos(logcos\alpha) + isin(logcos\alpha)]$$

   b) State and Prove Eulers Theorem for function of Three Variables.   (6)

   c) Expand $x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$ and   (8)
      hence find $f\left(\frac{11}{10}\right), f(0.99)$.

Q.6) a) Prove that:
$$sinh^7x = \frac{1}{64}(sinh7x - 7sinh5x + 21sinh3x - 35sinhx)$$ (6)

   b) Find nonsingular matrices P and Q such that PAQ is in Normal form.

      also find Rank of   A , where A=$\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$.   (6)

   c) Using Newton Raphson Method find an iterative formula for $\sqrt[5]{N}$ where
      N is positive number, Hence find $\sqrt[5]{35}$.   (8)

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