Q. P. Code : 25157

Duration: 3 Hours **Max. Marks 80**

N.B.

1. **Q.1** is compulsory. Attempt **any three** from the remaining questions.
2. All questions carry equal marks.
3. Figures to the Right indicate full marks.
3. Assume suitable data if necessary

Q.1 Attempt **any four** **20**

a. To determine the stability of the system $\dot{x} = Ax$, derive the Lyapunov equation $A^{\top}P + PA = -Q$ for symmetric positive definite P and Q.

b. Define the singular point in phase-plane. Compute the singular points for the following system.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 + x_2 + x_1^2\end{aligned}$$

c. Compute the 2-norm for the following matrices

$$\textbf{(i)}\ F = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix},\quad \textbf{(ii)}\ G = \begin{bmatrix} 1 & 0.5 \\ 0.2 & 3 \end{bmatrix}$$

d. Define relative degree for the system $\dot{x} = f(x) + g(x)u$ at $y = h(x)$.

e. Derive the classical control '**c**' from the IMC controller '**q**'.

f. What is describing function? What are the assumptions made to represent the nonlinearity with describing function.

Q.2 A. Design the IMC controller for the system model **10**

$$\tilde{G}_p = \frac{e^{-3s}}{25s+1}$$

to track the step input.

B. Obtain the IMC based PI controller for the model **10**

$$\tilde{G}_p = \frac{1}{5s+1}$$

Q.3 A. Construct the Lyapunov function using Krasovskii's method for the system, **10**

$$\begin{aligned}\dot{z}_1 &= z_2 - z_1^3\\ \dot{z}_2 &= -z_1 - z_2\end{aligned}$$

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B. Determine the stability of the system, **10**

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 - 2x_2\end{aligned}$$

using Lyapunov's method

Q.4 A. Obtain the describing function for Realy-with-deadzone nonlinearity. **10**

B. Linearize the following system using feedback control **10**

$$\begin{aligned}\dot{x}_1 &= x_1 + 2x_2^2 + e^{x_1}u\\ \dot{x}_2 &= x_1\\ y &= x_2\end{aligned}$$

Where y is output and u is input.

Q.5 A. Design the optimal control for the system **10**

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & 1.5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

that minimizes the performance index

$$J = \int_0^\infty \left\{ x^\top \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} x + 2u^2 \right\} dt$$

B. Construct the phase trajectory for the system $\ddot{x} + 1 = 0$ using delta method. Consider an initial condition $x(0) = 1,\ \dot{x}(0) = 1$. **10**

Q.6 A. Write the classification of singular points for second order linear systems with neat diagrams of representative pole locations and corresponding phase trajectory. **10**

B. Explain the limit cycles for Vander Pol's equation. **10**
