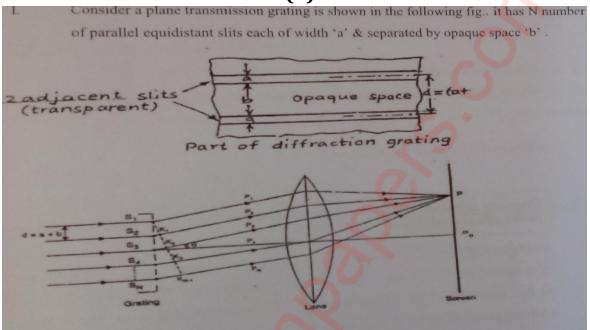
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#### MUMBAI UNIVERSITY CBCGS SEM II

#### APPLIED PHYSICS II JUNE 2023 PAPER SOLUTIONS

#### Q1.A. How The Condition For Absent Spectra In A Grating Is Obtained? (3)



П.	The intensity of light due to the diffraction grating in a direction making an angle $\theta$
	with the normal to the grating & at a point P is given by
	$l_{\theta} = l_m \left(\frac{\sin \alpha}{\alpha}\right)^2 \cdot \frac{\sin^2 N\beta}{\sin^2 \beta}$
Where	$\alpha = \frac{\pi}{\lambda}(a+b).\sin\theta \ \&\beta = \frac{\pi}{\lambda}(a+b).\sin\theta$
111.	The factor $I_m \frac{\sin^2 \alpha}{\alpha}$ gives the intensity distribution due to a single slit while the
	factor $\frac{\sin^2 N\beta}{\sin^2 \beta}$ gives that due to the combined effect if N slits.
	The principle maxima in the case of grating are obtained in the direction given by $(1)$
IV.	
	$(a+b)sin\theta$
have been a feature	Where, m= order of the spectrum
V.	The minima in the case of a single slit are obtain in the direction given by
	$\alpha \sin \theta = n\lambda \tag{2}$
VI.	If both the condition (1) & (2) are satisfied simultaneously, a particular maximum
¥ 1.	order 'm' will be missing in the grating spectrum. Therefore dividing equation (1) by
	(2) we get
	$\frac{a+b}{a} = \frac{m}{n}$
	a n
This is the condition for the absent spectra	
Highest possible orders condition for maxima in a diffraction grating is given by	
$(a+b)\sin\theta = \pm m\lambda$ or $m = \frac{(a+b).\sin\theta}{\lambda}$	
where m will be maximum when $\sin \theta = 1$ &	
$m_{max} = \left(\frac{a+b}{\lambda}\right)$	

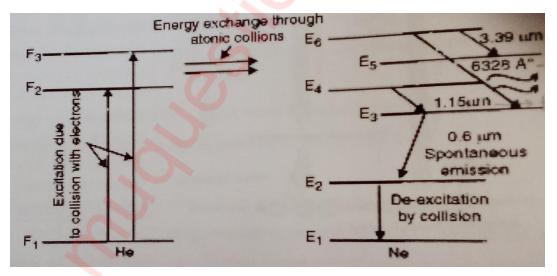
#### Q1.B Draw And Explain Energy Lvel Diagram For He-Ne Laser . What Is Role Of Helium Atoms? (3)

**Ans:** He-Ne Laser Consists Of A Long Narrow Discharge Tube Filled With He-Ne In The Ratio 10:1 With A Pressure Of 1mm Mercury. Voltage One Kilovolt Applied Across The Tube Slow Discharge Of The Gas Then Many Electrons Are Freed From The Gas Atoms Which Are Accelerated Towards Positive Electrode And Collide With H Therefore He Atoms Are Larger In Number Excitation Of He Takes Place .The He Atoms Are Excited Energy Level F2 And F3 Which Are Metastable States. The Population Increases Rapidly And Population Inversion Takes Place Between E6 And E4 With Respect To E5 And E3. E6 & E5: It Gives To A Radiation Of Wavelength 33,912 Armstrong Unit Which Is An Infrared Region Not Visible.

E6&E3: The Radiation Of Wavelength 6328 Armstrong Unit Which Is Visible And Of Red Color.

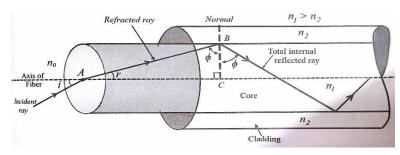
E4&E3: It Gives Rise To A Radiation Of Wavelength 11,523 Armstrong Unit Which Is Also An Infrared Region From E5 To E3 Levels Atoms Undergo Spontaneous Transition To E-2 Level At Much Faster Rate But E-2 Level Is Metastable For Ne The Atoms Will Come Down To Ground By Wall Collision It Is Kept Only Of Umm To Enable Efficient Depopulation Of E-2 Level Since The Discharge Is Continuous The Emission Of Laser Is Also Continuous Therefore He-Ne Laser Is Called As Continuous Wave Laser.

Role Of He In He- Ne Laser: Actually He Atoms Are Not Directly Involved In The Laser Action The Role Of He Is To Excite Neon Atoms And Cause The Population Inversion By Providing The Additional 0.05 Ev Due To The K.E Of He Atoms For Exciting The Ne Atom To E6 Level. Helium Also Impressive The Conduction Of Heat To The Walls Of Tube.



Q1.C With The Help Of The Diagram Define The Term Acceptance Angle. (3)

Ans: It Is Defined As The Maximum Angle That An Incident Light Rays Gets Back With Respect To The Fiber Axis And Propagates The Fiber. Mathematically It Can Be Expressed As



#### Q1.D If $\Phi(X,Y,Z)=3x^2y-Y^3z^2$ , Find $\Phi$ at The Point (-1,-2,1) (3)

#### Ans:

The gradient is  $= \langle -12, -9, -16 \rangle$  **Explanation:** The gradient is a vector :  $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$   $f(x, y, z) = 3x^2y - y^3z^2$   $\frac{\partial f}{\partial x} = 6xy$   $\frac{\partial f}{\partial y} = 3x^2 - 3y^2z^2$   $\frac{\partial f}{\partial z} = -2y^3z$   $\nabla f(x, y, z) = (6xy, 3x^2 - 3y^2z^2, -2y^3z)$  $\nabla f(1, -2, -1) = (-12, -9, -16)$ 

#### Q1.E Calculate The Velocity Of A Particle Having Kinetic Energy Three Times Its Rest Mass Energy. (3)

Ans: We Know M =  $[Mo / \sqrt{\{1 - (V2 / C2)\}}]$ Given: M = 3 Mo Hence 3mo =  $[Mo / \sqrt{\{1 - (V2 / C2)\}}]$  $\therefore 1 - (V2 / C2) = (1/9)$  $\therefore (V2 / C2) = (8/9)$  $\therefore V = C \times (\sqrt{8} / 3)$ = 3 × 108 × ( $\sqrt{8} / 3$ )  $=\sqrt{8} \times 108$ 

 $:: V = 2.82 \times 108 \text{ M/S}$ 

#### Q1.F. Explains With An Example To Significance Of Surface Area To Volume Ratio In Nanotechnology. (3)

Ans:

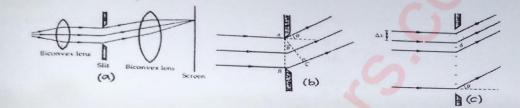
- The Surface Area To Volume Ratio For A Material Or Substance Made Up Of Nanoparticles Has A Significant Effect On Properties Of The Material.
- Nanoparticles Have A Relative Largest Office Area When Compared To The Same Volume Of Materials Made Up Of Bigger Particles.
- Let Us Consider A Sphere Of Radius Or The Surface Area Of The Sphere Will Be  $4\pi r^2$  To The Volume Of The Sphere =  $4/3\pi r^3$  Therefore The Surface Area To The Volume Ratio Will Be
- $4\pi r^2/(4/3\pi r^3)=3/R$ .
- It Means That The Surface Area To The Volume Ratio Increases As The Radius Of The Sphere Decreases And Vice Versa. It Also Means That When A Given Volume Of A Material Is Made Up Of Smaller Particles The Surface Area Of The Material Increases.
- Materials Made Of Nanoparticles Have A Much Greater Surface Area Per Unit Volume Ratio Compared With The Materials Made Up Of Bigger Particles This Leads To Nanoparticles Being More Chemically Reactive.

# Q2.A Discuss With Appropriate Diagram The Phenomena Of Fraunhofer's Diffraction At A Single Slit And Write The Conditions For Its Maxima And Minima. (8)

Ans: The Phenomena Bending Up Light Around The Corner Of An Opaque And Spreading Of Light Waves Into The Region Of Geometrical Shadow Of The Obstacle Whose Dimensions Are Comparable With Wavelength Of Incident Light Is Called Diffraction.

In Fraunhofer Diffraction Light Source And The Screen Are At Infinite Distance From The Diffracting Aperture. Consider A Parallel Beam Of Monochromatic Light Of

Wavelength  $\Lambda$  Incident On A Slit Of Width 'A' Defracted At An Angle As Shown In The Figure:



The path difference between extreme rays diffracted from the slit is  $\Delta = BC = AB \cdot \sin \theta = a \cdot \sin \theta \text{ and}$ 

the phase difference between these rays is given as follows

$$\phi = \frac{2\pi}{\lambda},$$
$$\Delta = \frac{2\pi}{\lambda} a \sin \theta$$

The slit is now divided into 'N' parts of equal width ' $\Delta X$ ' as shown in fig. Since width ' $\Delta X$ ' is very small, each part behaves like a point source. As all parts are of equal width, the amplitude ' $\Delta E$ ' of the waves transmitted by them will be the same.

The path difference between waves transmitted by two adjacent parts is  $\delta = \Delta x$ . sin  $\theta$  therefore the phase difference is

 $\Delta \phi = \frac{2\pi}{\lambda} \Delta . \sin \theta$ .

#### **Condition For Minima :**

the minimum intensities occur at the angles given by,

$$\theta = \sin^{-1}\left(\frac{n\lambda}{\alpha}\right); n = \pm 1, \pm 2, \pm 3 \text{ where } \theta =$$

 $\pm \pi, \pm 2\pi, \pm 3\pi$ 

#### **Condition For Maxima:**

$$\frac{I_{\theta}}{I_m} = \left[\frac{\sin\left(n + \frac{1}{2}\right)\pi}{\left(n + \frac{1}{2}\right)\pi}\right]^2$$
  
Butting n=1, 2, 3..., we get  $\frac{I_{\theta}}{I_m} = 0.0450, 0.0162, 0.0083$ 

Q2.B With Neat And Labeled Diagrams Explain The Construction And Working Of A Semiconductor Laser. Give Its Application. (7)

Ans: A Semiconductor Diode Laser Is Specially Fabricated Cn Junction Device That Emits Coherent Light When It Is Forward Biased.

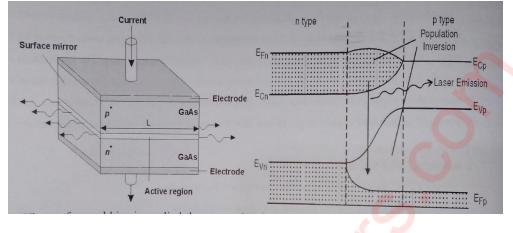
Construction:

The Diodes Is Small And The Junction And Lies In A Horizontal Plane Through The Center. The Top And Bottom Faces Are Metalised And Omic Contact Are Provided To Pass The Current In The Diode. The Front And Rear Faces Are Polished Parallel To Each Other And Perpendicular To The Plane Of The Junction .These Faces Form The Resonator .The Other Two Opposite Faces Are Rough And To Prevent Lazing In That Direction .The Active Region Consists Of A Layer Of About 1 Mm Thickness It Is Heavily Built P-N Junction. Working:

When The Junction Is Unbiased Of Fermi Level Is Uniform Across The Junction At Low Forward Of Current The Electron Hole Recombination Causes Spontaneous Emission Of Photons And The Junction Linearly When The Current **Reaches A Threshold Value The Carrier Concentration In The** Deletion Region Reaches Very High Values With Electrons In The Conduction Band And Holes In The Valence Band The Upper Levels In The Depletion Region Have High Population Density Of Electrons Then The Lower Levels In The Same **Region Are Vacant.** This Is The State Of Population Inversion Or Active Region Therefore The Forward Biased Current Act As A Pumping Agent In Semiconductor Diode Laser. The Photons That Propagate In The Junction Induce The Conduction Electrons To Jump Into The Vacant States Of Valence Band. The Cavity Reflect The Photons To The Active Medium.

Applications:

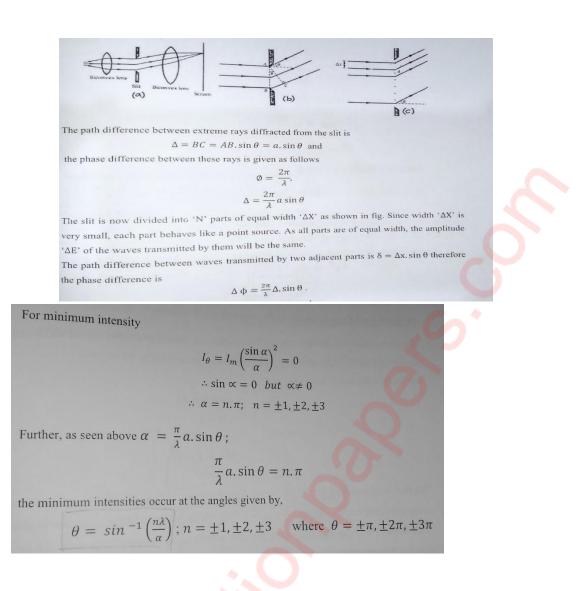
Semiconductor Diode Laser Is Used In Laser Printers And Copiers, Cd Player Optical Communication (As A Light Source).



## Q3.A Discuss The Phenomena Of Fraunhofer's Diffraction At A Single Slit And Obtain The Condition For First Minimum. (8)

Ans: The Phenomena Bending Up Light Around The Corner Of An Opaque And Spreading Of Light Waves Into The Region Of Geometrical Shadow Of The Obstacle Whose Dimensions Are Comparable With Wavelength Of Incident Light Is Called Diffraction.

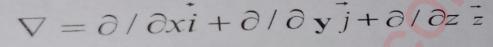
In Fraunhofer Diffraction Light Source And The Screen Are At Infinite Distance From The Diffracting Aperture. Consider A Parallel Beam Of Monochromatic Light Of Wavelength Λ Incident On A Slit Of Width 'A' Defracted At An Angle As Shown In The Figure:



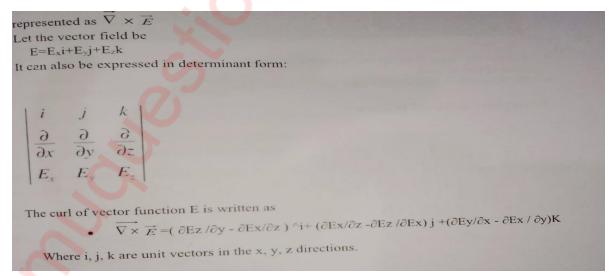
#### Q3.B What Are Scalar And Vector Fields? How Is A Del Operator Expressed? Explain The Term 'Curl Of A Vector And States Its Significance'. Show That The Divergence Of The Curl Of A Vector Is Zero. (7) Ans:

 Scalar Field Associated A Scalar Value To Every Point In Space It May Be Representing Either A Mathematical Number Or Physical Quantity. Eg: Pressure Distribution In A Fluid.

- A Vector Field Is Specified By Both The Magnitude And Direction Of A Physical Quantity At Each Point Of The Field Region.
- The  $D_{\vec{v}}$  Operator :
- It Is A Vector Differential Operator Which Is Essential For The Study Of Electrodynamics. It Is The Collection Of Partial Derivative Operators.



- This Is Commonly Called The Del Operator. It Is Not Vector In Itself But When Operates On Scalar Function It Provides The Resultant As Vector It Can Be Operated In Differential Ways.
- Curl Is An Operation Done On Vector Field The Divergence Of Vector Field Produces A Vector The Curl Of A Vector Function Is The Vector Product Of The Del Operator With A Vector Function. Curl Is Simply Defined As Circulation For Unit Area Where Closed Path Vanishingly Small.



Physical Significance Of Curl: Curl Is Measure Of How Much Of Vector Curls Rotates Around A Point In Question. The Curl Of A Vector Field Measures The Tendency For The Vector Field To Swirl Around. Imagine That The Victor Field Represents The Velocity Vectors Of Water In A Lake .If The Vector Field Swirls Around Then We Stick A Paddle Wheel Into The Water It Will Tend To Spin The Direction Of Curl Is The Axis Of Rotation As Determined By The Right Hand Rule And The Magnitude Of Curl Is The Magnitude Of Rotation.

Divergence Of Curve Is Zero:

For any field 
$$\vec{B}$$
 the divergence of a curl of  $\vec{B}$  is written as  
 $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \left(\hat{i}_x \frac{\partial}{\partial x} + \hat{i}_y \frac{\partial}{\partial y} + \hat{i}_z \frac{\partial}{\partial z}\right)$ .  
 $\left[\hat{i}_x \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{i}_y \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{i}_z \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\right]$   
 $= \frac{\partial}{\partial x} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \frac{\partial}{\partial y} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \frac{\partial}{\partial z} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$   
 $= 0$ 

Q4.A What Do You Understand By Resolving Power? How
Can The Resolving Power Of A Grating Be Increased?
Find Maximum Resolving Power Of A Grating Of Width 7
Centimeters Illuminated By A Laser Beam Of Wavelength
4800 Armstrong Unit Having 1000 Lines Per Centimeter.
(5)

Ans:

- According To Rayleigh's Criterion Two Closely Space Point Source Of Light Are Said To Just Resolve By An Optical Instrument Only If The Central Maximum In The Diffraction Pattern Of One Falls Over The First Minimum In The Diffraction Pattern Of The Other And Vice Versa.
- R.P= $\Lambda$ /D $\lambda$ = M.N

Resolving Power Of The Grating Can Be Increased By

Using The Incident Light Of Higher Wavelengths.

- Considering Higher Order Spectrum.
- Increasing N That Is Total Number Of Slit.
- Width W = 7 Cm, N = 1000 Lines/Cm, Λ = 4800 A° Grating Element (A+B)=1/N=1/1000

Rp Is Maximum When N Is Maximum

N Is Maximum When Sin  $\Theta$  =1

(A+B) Sin  $\Theta$  = N  $\Lambda$ 

 $Nmax=1/N\lambda=1/1000 \times 4800 \times 10^{-8}=4.8$ 

N Cannot Be 7 As Sin  $\Theta$  Exceeds One

Nmax = 4.8

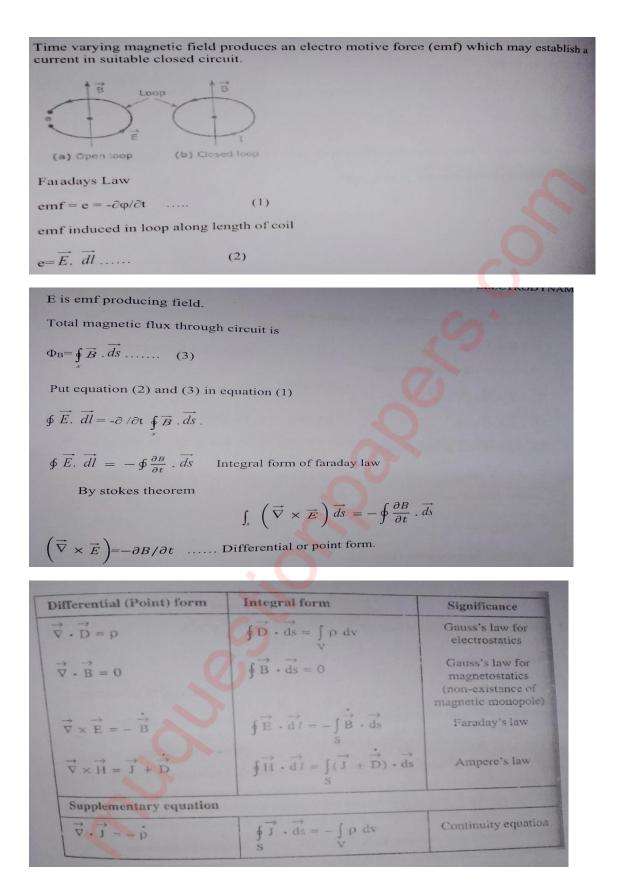
Rp = N N

Rp = 4.8x1000 X 7 As Grating Is 7 Cm Wide

Rp = 33600

#### Q4.B State And Derive Maxwell's Equation Which Describes How The Electric Field Circulates Around The Time Varying Magnetic Field(Differential Form). (5)

Ans: Maxwell's A Set Of Equation Form The Foundation Of Electromagnetic Theory Or Extension Of Work Of Gauze For A Day And Ampere Maxwell Equation Are Classified Into Two Categories For Static Fields And Time Varying Fields 2<sup>nd</sup> In Differential Form And In Integral Form.



Q4.C A Step Index Fiber Has A Core Diameter Of 33 X10-6 M. The Refractive Indices Of Core And Cladding Are 1.56 And 1.5189 Respectively. If The Light Of Wavelength 1.3 μm Is

## Transmitted Through The Fiber, Determine Normalized Frequency Of The Fiber. Weather Fiber Supports Single Mode Or Multimode. (5)

## Ans: <u>Given,</u>

For A Step-Index Fiber: Core Diameter =  $33 \times 10^{-6}$  M, Refractive Index Of Core = 1.56, Refractive Index Of Cladding = 1.5189, Wavelength Of Light = 1.3 Mm.

## <u>TO Find,</u>

Normalized Frequency,

The Number Of Modes The Fiber Will Support.

## <u>Solution:</u>

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Where,

A = Core Radius,  $\Lambda$  = Light Wavelength, N<sub>1</sub> = Refractive Index Of Core, N<sub>2</sub> = Refractive Index Of Cladding. **Core Radius = A = 16.5 × 10<sup>-6</sup> M**,  $\Lambda$  = 1.3 × 10<sup>-6</sup> M, N<sub>1</sub> = 1.56, N<sub>2</sub> = 1.5189. Substituting All Values In Equation 1, We Get, V=2.835 Number Of Modes = V<sup>2</sup>/2 Therefore, (28.35)<sup>2</sup>/2 = 4.0186 ...(1)

#### Q5.A Explain Gauss's Laws For Static Electric And Static Magnetic Fields In Differential And Integral Form. (5)

is Electric flux density Q is the total charge enclosed by closed surface. This may be expressed volume integral of charge density  $\rho$ Gauss law is written in integral form  $\Phi = \oint \vec{D} \cdot ds = Q(enclosed)$  $\oint D \, ds = \int \rho \, dv \, ----1)$ Applying divergence theorem  $\oint \overrightarrow{D} \cdot ds = \int (\overrightarrow{\nabla} \cdot \overrightarrow{D}) dv \cdots 2 \rho$ Equation (1) becomes  $\int \left( \overrightarrow{\nabla} \cdot \overrightarrow{D} \right) dv = \int \rho \cdot dv \dots 3$ Hence Gauss law electric field in differential form or point form  $\vec{\nabla} \cdot \vec{D} = \rho$ ,  $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon 0}$  $\vec{D} = \varepsilon \vec{D}$ In an Electric and magnetic field, any closed surface, real/Imaginary is called Gaussian surface. In a magnetic field the magnetic lines, the magnetic lines are closed on themselves. Hence total outgoing flux is zero.

$$\oint \vec{B} \cdot ds = 0$$

This is called Guass law for magnetic field in integral form.

Using divergence theorem, Gauss law can be written as

 $\oint \vec{B} \cdot \vec{ds} = \oint (\nabla \cdot B) \quad d\nu = 0 \quad \text{Or} \quad \nabla \cdot B = 0$ 

This is called differential or point form.

#### Q5.B Explain The Concept Of Relativity. Comment On Galilean And Lorentz Transformations (5)

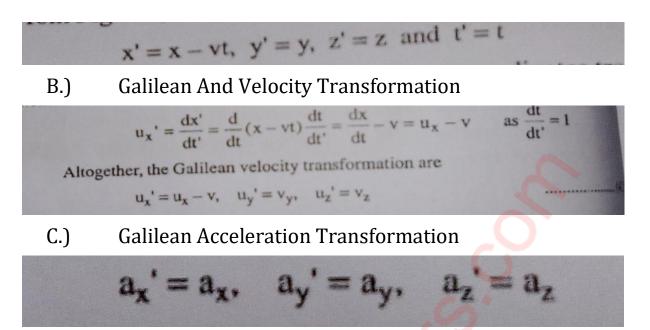
Ans: The Theory Of Relativity Usually Encompasses Two Interrelated Theories By Albert Einstein Special Relativity And General Relativity Special Relativity Applies To All Physical Phenomena In The Absence Of Gravity General Relativity Explains The Law Of Gravitation And Its Relation To Other Forces Of Nature.

The Theory Of Special Relativity Explains How Space And Time Are Linked For Objects That Are Moving At A Consistent Speed In A Straight Line 1 Of Its Famous Aspects Concerns Object To Moving At The Speed Of The Light.

The Transformation From One Inertial Frame Of Reference To Another Is Called Galilean Transformation Knowing The Laws Of Motion Of An Object In A Reference System S The Laws Of Motion Of The Same Object In Another Reference System S' Can Be Derived.

Galilean Transformation Has Been Segregated Among Three Types Coordinate Transformation Velocity Transformation And Acceleration Transformation.

A.) Galilean Coordinate Transformations.



#### Q5.C What Is Nano Material? Explain Any One Method Of Production Of Nano Material. (5)

Ans: Nanomaterials Are The Materials Where The Atom Size Ranges Between One To Hundred Nanometers Further The Atoms Do Not Make Each Other The Technology Evolved Out Of This Is Called Nano Technology. Navy Materials Have Properties That Are Different From Those Of Bulk Materials.

There Are Many Methods Of Production Of Nano Material One Of Them Is Sputtering.

Sputtering On Laser Operation May Be Used Instead Of Thermal Evaporation Sputtering Doesn't Non Thermal Process Index Surface Atoms Of Physically Ejected From The Surface By Momentum Transfer From An Energetic Bombarding Species Of Atomic Or Molecular Size Typical Sputtering User Glow Discharge Or Iron Blimp Interaction Events Which Occur At Near The Target Surface During The Sputtering Process In Magnetron Sputtering Has Advantage Over Diode And Tridiode Sputtering.

#### Q6.A State And Explain Application Of Optical Fibre With Suitable Example. (5)

Ans: An Optical Fiber Is A Flexible Transparent Fiber Made Of Extruded Glass Silica Or Plastic Slightly Thicker Than A Human Hair. It Can Function As A Wave Guide On Light Pipe To Transmit Light Between The Two Ends Of The Fiber.

There Are Many Applications Of Optical Fiber Some Are Listed Below:

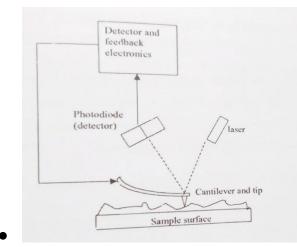
- 1.) Communication System The Fiber Optic Communication System Involves A Transmission Of Information From The Source To The Destination In The Form Of Pulses Of Infrared Light Signals.
- 2.) Medicine Optical Fibers Are Widely Used In Various Medical Equipment And Research Laboratory Machines For Instance The Endoscope Autoscope Of The Ophthalmoscope.
- 3.) Sensors Like Light Emitting Diodes Or Laser Light Up Preferred Sources For Transmitting Signals In Optical Fiber Communication.
- 4.) Internet Being One Of The Prominent Example Optical Fibers Can Transmit A Large Amount Of Data From One Place To Another In Relatively Less Time.

## Q6.B Explain Construction And Working Of Atomic Force Microscope (5)

Ans: Construction And Working

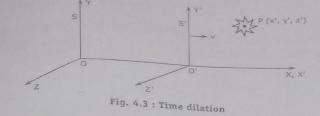
- It Is A Modified Tem In Which The Limitation Of Tem Are Overcome You're The Noodle Tip Is Kept In Touch With The Sample Keeping The Force In The Tips Constant The Scanning Is Done Clearly The Tip Will Have Vertical Movements Which Depend Upon The Topography Of The Sample.
- The Tip Has A Mirror On The Top Of It A Laser Beam Is Used To Have The Record Of Vertical Movement Of The Needle This Information Is Later Converted To Visible Form Using A Photo Diode As Afn Does Not Generate Any Current The Problem Of Non Conducting Sample Material Is Overcome.
- 3d Dimensional Surface Profile Can Be Obtained It Can Be Used In Air Or Even With Liquid But Scanning Speed Is Low

And As The Tip Is In Contact With The Sample It Is Likely To Desorted The Biological Sample.



#### Q6.C What Is Time Dilation And Space Contraction Using Lorentz Transformations Obtain Expression For Them. (5)

Ans: The Meaning Of Time Dilation Is Extension Of Time Dilation Is A Difference In The Elapsed Time Measured By Two Blocks Due To A Relative Motion Between Them To Explain It Let Us Consider 2 Frames Of Reference S And S' With S' Moving With A Velocity V Along X Direction With Respect To S' As Shown In Figure Imagine A Gun Placed At A Fixed Position In The Frame Suppose It Fires Two Shorts At Instance T1' And T2' Measured By The Observer O' In The Frame S'.



The time interval  $(t_2' - t_1')$  of the two shots measured by O' at rest in the moving frame S' is called the proper time interval and is given by

$$T_0 = t_2' - t_1'$$
 (4.15)

As the motion between the two frames is relative, we may assume that the frame S is moving with velocity -v along the -X direction relative to frame S'. In frame S, the observer O who is at rest hears these two shots at different times  $t_1$  and  $t_2$ .

The time interval appears to him is given by

t =

$$t_2 - t_1$$

From inverse Lorentz transformation equations, we get

$$t_{1} = \frac{t_{1}' + (vx'/c^{2})}{\sqrt{1 - (v^{2}/c^{2})}}$$

$$t_{2} = \frac{t_{2}' + (vx'/c^{2})}{\sqrt{1 - (v^{2}/c^{2})}}$$
(4.17)
(4.18)

Substituting equations (4.17) and (4.18) in equation (4.16), we get

 $T = \frac{T_o}{\sqrt{1 - (v^2/c^2)}}$ 

$$\Gamma = \frac{t_2' - t_1'}{\sqrt{1 - (v^2/c^2)}}$$
(4.19)

Using equation (4.15) in equation (4.19), we have

...... (4.16)

which shows that  $T > T_0$ .

Here, T<sub>o</sub> is called the proper time which is defined as the time measured in the frame of reference in which the object is at rest.

This verifies that the actual time interval in the moving frame appears to be lengthened

by a factor  $\frac{1}{\sqrt{v^2 - c^2}}$  when it is measured by an observer in the fixed frame, v being the relative velocity between the two frames.