1T01033 - S.E.(Electronics and Telecommunication )(SEM-III)(Choice Base Credit Grading System ) (R- 19) (C Scheme) / 51221 - Engineering Mathematics-III

QP CODE: 10037953 DATE: 21-11-2023

**Time: 3 hour** **Max. Marks: 80**

**Note: 1) Question 1 is compulsory.**

**2) Attempt any 3 questions from Question 2 to Question 6**

**3) Figures to the right indicate full marks.**

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|---|---|---|
| **Q1** | **Attempt All questions** | **Marks** |
| **A** | If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ then find the eigen values of $A^{-1} + A^2$ | **5** |
| **B** | Find Laplace transform of $f(t) = t\{\sqrt{1 + sint}\}$ | **5** |
| **C** | Find the Fourier Series for $f(x) = x^2$ , where $x \in (-\pi, \pi)$ | **5** |
| **D** | Prove that $f(z) = logz$ is analytic, also find its derivative. | **5** |
| **Q2** | | |
| **A** | Using Green's theorem in a plane to evaluate $\oint_C (x^2 - y^2)dx + (x + y)dy$ and C is the triangle with vertices (0, 0) , (1, 1) and (2, 1) | **6** |
| **B** | Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ | **6** |
| **C** | Show that the function u = sinx coshy + 2cosx sinhy + $x^2 - y^2 + 4xy$ satisfies Laplace's equation, also find analytic function. | **8** |
| **Q3** | | |
| **A** | If $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ show that $\overline{F}$ is irrotational and solenoidal. | **6** |
| **B** | If v = $e^x siny$ , prove that v is a harmonic function. Also find the corresponding harmonic conjugate. | **6** |
| **C** | Prove that the matrix A is diagonalisable, also find diagonal form and transforming matrix. $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ | **8** |

**Q4**

**A** Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ **6**
Where $\bar{F} = x^2 i - xyj$ and C is the square in the plane z = 0 and bounded by x = 0, y = 0, x = a and y = a

**B** Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$, using Laplace transforms **6**

**C** Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ **8**

**Q5**

**A** Find $L\left\{\int_0^t u \sin 4u \, du\right\}$ **6**

**B** Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by **6**
$$\bar{F}(x,y,z) = y\,\hat{\boldsymbol{i}} + (z\cos(yz) + x)\,\hat{\boldsymbol{j}} + (y\cos(yz))\,\hat{\boldsymbol{k}}$$
Show that $\bar{F}$ is conservative and find its scalar potential.

**C** Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$ where **8**
$$f(x) = x + \frac{\pi}{2} \quad -\pi \le x \le 0$$
$$= \frac{\pi}{2} - x \quad 0 \le x \le \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$

**Q6**

**A** Obtain Fourier series expansion of $f(x) = 4 - x^2$ in (-2, 2) **6**

**B** Verify Cayley-Hamilton theorem for the matrix A and hence find $A^{-1}$ and $A^4$ where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **6**

**C**

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{s+a}{s+b}}\right)\right\}$ **4**

ii) Find $L^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$ **4**

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